Introduction to abelian anyons in the F.Q.H.E.

Francesco Mancarella- SISSA,Trieste

SISSA-Trieste, Italy

June 3, 2010

Reference:

"Anyons and the quantum Hall effect- A pedagogical review", Ady Stern, Annals of Physics, January 2008

Outline

- Goals of the discussion
- Quantum Hall Effect (Q.H.E.)
- Aharonov-Bohm effect
- Fractional charged excitations (combining Q.H.E. and Aharonov-Bohm effect)
- Charges and Vortices- Lorentz and Magnus forces in superfluids and superconductors
- Quasi-holes in the $\nu-$F.Q.H.E. are $\pi\nu-$anyons !
- Interferometry as a way to observe anyons: Fabry-Perot interferometer.
- Some experimental difficulties

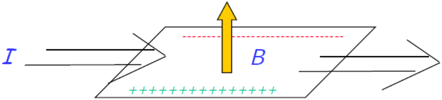
Introduction

- system confined to $(2+1)D \longrightarrow$ bosons, anyon, or fermions
- Our goals:
  - describe the Q.H.E.
  - explain why the observation of F.Q.H.E. implies the notion of anyons
  - show possible ways for a direct observation of the anyonic physics
- to describe how the physics of Anyons enters in the Q.H.E., we will consider: the Q.H.E., the Aharonov-Bohm effect, the Berry geometric phase

The Quantum Hall effect

Introduction to the classical and quantum Hall effect

Electrons in two dimensions

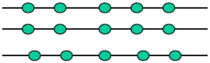


Classically,

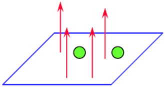
Hall resistivity - $\dfrac{V_y}{I_x} = \dfrac{B}{nec}$

longitudinal resistivity - unchanged by *B*.

Quantum mechanically degenerate harmonic oscillator spectrum



$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$ Landau levels



Landau level filling factor =

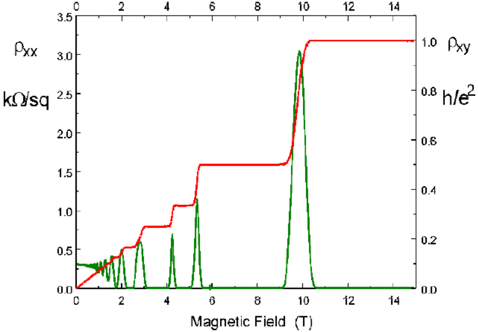
$\nu \equiv \dfrac{\text{density of electrons}}{\text{density of flux quanta}}$ $\Phi_0 = hc/e$

The quantum Hall effect

- zero longitudinal resistivity - no dissipation, bulk energy gap

current flows mostly along the edges of the sample

- quantized Hall resistivity



$$\rho_{xy} = \frac{1}{\nu}\frac{h}{e^2}$$

ν is an integer,

or a fraction p/q with q odd,

or q even

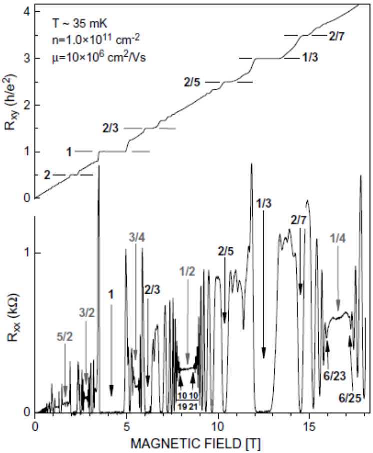
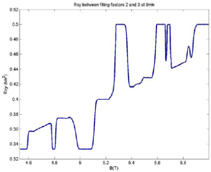


Fig. 1. The quantum Hall effect. When the Hall resistance is measured as a function of magnetic field plateaus at quantized values are observed. In regions of the magnetic field where the Hall resistance is in a plateau, the longitudinal resistance vanishes (Sample grown by L.N. Pfeiffer (Lucent-Alcatel) and measured by W. Pan (Sandia)).

The Landau filling range of $2<\nu<4$

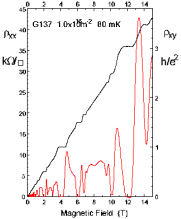
Unconventional fractional quantum Hall states:

1. Even denominator states are observed $\nu = \frac{5}{2}, \frac{7}{2}, \frac{19}{8}$
2. Observed series does not follow the $\nu = \frac{p}{2p+1}$ rule.
3. In transitions between different plateaus, ρ_{xy} is non-monotonous



as opposed to

(Pan et al., PRL, 2004)



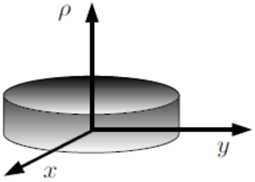
Focus on $\nu=5/2$

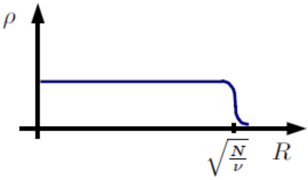
- Within a certain range of temperatures, the deviation of σ_{xx}, σ_{xy} from their $T=0$ values follow the **activation law**: $\propto e^{-T/T_0}$, T_0 temperature scale depending on the details. This is a fingerprint of the presence of a gap between ground and first excited states.

Laughlin's quantum incompressible fluid

Electrons form a droplet of fluid:

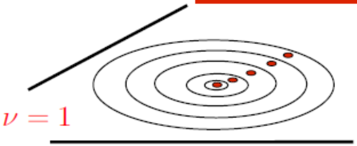
- incompressible = gap
- fluid = $\rho(x,y) = \rho_o = \text{const.}$

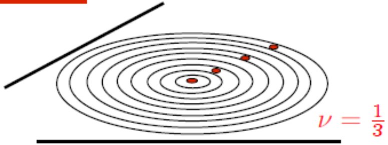




$\mathcal{D}_A = BA/\Phi_o$, # degenerate orbitals = # quantum fluxes, $\Phi_o = \frac{hc}{e}$

filling fraction: $\nu = \frac{N}{\mathcal{D}_A} = 1, 2, \ldots \frac{1}{3}, \frac{1}{5}, \ldots$ density for quantum mech.





The Aharonov-Bohm effect

- System of electrons in an e.m. field. The force results from one term in the action, that is GEOMETRIC if the vector potential is independent of time:

$$\frac{e}{c}\int dtv(t)\cdot A(x,t) = \frac{e}{c}\int dl \cdot A(x)$$

This action (divided by $\hbar$) gives the phase of the contribution of that trajectory to the propagator of the particle:

$$\textbf{Aharonov-Bohm}\text{ phase} = 2\pi\frac{\Phi}{\Phi_0},$$

$\Phi_0 = \frac{hc}{e}$ flux quantum.

Two experimental sets-up to think about the Aharonov-Bohm effect:

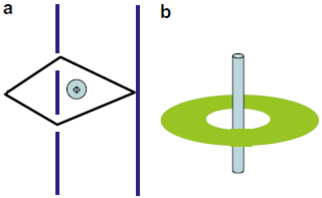


Fig. 2. The Aharonov–Bohm effect. Two realizations of the Aharonov–Bohm effect. In the first, shown in (a), the magnetic flux induces a shift in the interference pattern of a double slit experiment. An analog effect is to be seen in the quantum Hall interferometers discussed in Sections 7 and 12. In the second, shown in (b), the magnetic flux affects the thermodynamic properties of the annulus around it. This realization is very useful in the analysis of the fractional charge and fractional statistics of quasi-particles in the quantum Hall effect, see Section 4.

(Persistent current observed in mesoscopic rings)

Excitations of fractional charge: combining A-B effect and Q.H.E.

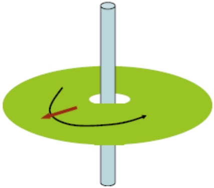


Fig. 3. The gedanken experiment to create an eigenstate of a fractional charge, later realized to be an anyon. The electrons on the annulus are in a Laughlin fractional quantum Hall state of filling fraction $\nu = 1/m$. The introduction of a flux quantum into the hole pushes a fraction of an electron charge from the interior to the exterior, leaving the system in a eigenstate.

- The total charge displacements results to be $Q(t) = \frac{e^2\nu}{hc}\Phi(t)$
  When $\Phi = \Phi_0$ (same spectrum of $\Phi = 0$): $Q = e\nu$
- For the adiabatic theorem, the system has to remain in an eigenstate during the turning on of the flux $\rightarrow$
  radial charge tunnelling requires a **breaking of rotational invariance** $\rightarrow$
  IMPURITIES .

- Laughlin's proposal for the wave function of a QUASI-HOLE at the origin:

$$(\prod_i z_i)\,\psi_{g.s.} = (\prod_i |z_i|)e^{i\sum_i \phi_i}\psi_{g.s.},$$

where

$$\psi_{g.s.}(\{z_i\},\{\bar{z}_i\}) = \prod_{i<j}(z_i - z_j)^m \prod_i e^{-|z_i|^2/4l_h^2}, \quad l_h^2 \equiv \frac{\hbar}{eB}$$

(**Laughlin's ansatz for the ground state** of the $\nu = 1/m$-FQH state). The product $e^{i\sum_i \phi_i}$ gives each electron a phase $e^{i\phi_i}$ (Aharonov-Bohm phase), hence an angular momentum $\hbar$.

- So, **quasi-hole** states carry a **fractional charge** $e\nu$ and **one single quantum of vorticity**, localized at the interior edge of the annulus.

Charges and Vortices- Lorentz and Magnus forces in superfluids and superconductors

- For vortex in a neutral superfluid,

$$F_{Magnus} = 2\pi n l v \times \hat{z}.$$

- Like the Lorentz force, the Magnus force is proportional and perpendicular to the velocity of the vortex.
- From a quantum point of view, interference effects due to the motion of a vortex can be brought back to the A.-B. effect:
  - Classical Lorentz force $\rightarrow$ quantum A.-B. phase due to the motion of an electron $= \frac{e}{\hbar c} \int ds \cdot B$
  - Classical Magnus force $\rightarrow$ quantum phase due to the motion of a vortex $= \frac{2\pi l}{\hbar} \int ds \cdot n$
- Consequence: the phase shift in interference associated to a closed loop of a vortex is equal to <u>2π times the number of fluid particles encircled by the vortex as it goes around the loop</u>.

- for 2D charged superconductor the force is the same, but instead of being the hydrodynamical Magnus force, it's a Lorentz force:

$$F_{Lor} = \frac{1}{c}\int ds(J \times B) = \frac{1}{c}2ne\frac{\Phi_0}{2}v \times \hat{z} = 2\pi nlv \times \hat{z}$$

Analog of the Aharonov-Bohm effect for a VORTEX:

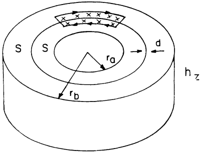


FIG. 1. The Josephson junction considered. Two concentric superconducting cylinders are separated by a thin insulating layer. The junction encloses a fluxon of magnetic field, formed by a "solenoid" of current.

- A vortex doesn't accumulate any phase in its wave function, by encircling another vortex.

Instead this phase exists for quasi-holes in the Q.H.E.

- Vortices in (charged or not) superfluids, and quasi-holes in Q.H. systems are collective excitations.

$\Psi(R) =$ wave function of a system with a vortex at R.

Dynamic of the vortex: given by the GEOMETRIC Berry vector potential

$$A_b = \Im \langle \Psi(R) | \nabla_R | \Psi(R) \rangle.$$

In terms of the fluid density ρ it becomes:

$$A_b = \int dr \rho(r) \frac{\hat{z} \times (r - R)}{|r - R|^2},$$

$\longrightarrow$ the vortex sees the fluid particles as an electron sees flux tubes.

• In both cases:

the phase accumulated by the wave function is 2π the expectation value of the number of FLUID PARTICLES enclosed in the loop.

QUASI-HOLES in F.Q.H.E. follow FRACTIONAL STATISTICS

Quasi-holes at Laughlin fraction $\nu = \frac{1}{m}$ carry charge $\frac{1}{m}e$, and ONE quantum of vorticity.

- Quasi-hole moving in the Q.H.fluid $\Rightarrow \vec{F} \perp \vec{v}$, $\vec{F} = 2\pi n l \vec{v} \times \hat{z}$, resulting from the vector potential $A_b = \int dr \rho(r) \frac{\hat{z}\times(r-R)}{|r-R|^2}$
- Constrain $n\Phi_0/B = \nu \Rightarrow \vec{F}$ = Lorentz force acting on a charge $e\nu$ moving in a magnetic field B with velocity v.
- $\Rightarrow$ we are brought to accept quasi-holes as collective degrees of freedom carrying a fractional charge $e\nu$.
- Phase accumulation exists for encircling among Q.H. quasi-holes because they correspond to fluid particle!
- Density of fluid particles such that the filling factor deviates from $\frac{1}{m} \Rightarrow$ **the deviation is accommodated in integer number of quasi-particles/ quasi-holes!**
- Quasi-holes are $\pi\nu$-phase ANYONS!

Extending the notion of quantum statistics

Halperin
Arovas et al

Electrons

Laughlin quasi-particles

A ground state: $\psi(r_1,\ldots\ldots\ldots\ldots\ldots r_N;\ R_1,\ldots,R_4)$

Energy gap

For abelian states:

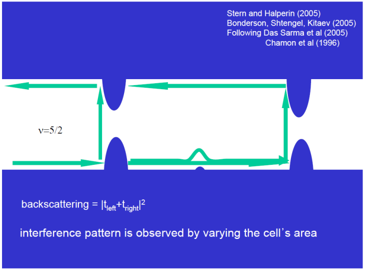
Adiabatically interchange the position of two excitations

$$\psi \longrightarrow e^{i\theta}\psi$$

INTERFEROMETRY AS A WAY TO OBSERVE ANYONS

Fabry-Perot interferometer

A Fabry-Perot interferometer:



t_1, t_2 tunnelling amplit. associated to the quantum points: at the lowest order:

$$\text{back-scattered current } I_{bs} \propto \left| t_1 + t_2 \right|^2$$

- 3 ways to modify the relative phase between t_1, t_2: vary B, n_0, S.
  - Grossly, a variation of S doesn't modify the filling factor ν, so it DOESN'T introduce quasi-particles in the bulk.
  - Instead a variation of n_0 or B introduces localized **quasi-holes** or **quasi-particles**

Simplest case: $\nu = 1$:

Aharonov -Bohm effect $\Rightarrow$ relative phase $= 2\pi$ times the number BS/Φ_0.

$$\Rightarrow I_{bs} \text{ sinusoidal in } S, \text{ with period } \Delta S = e/n_0.$$

$$\Rightarrow I_{bs} \text{ sinusoidal in } B, \text{ with period } \Delta B = \Phi_0/S.$$

Case of fractional Q.H.states, with filling factors $\nu = 1/m$:

We will consider only the tunnelling of quasi-particles (with charge $e\nu$).

For a quasi-particle encircling the cell between the two point contacts:

$$\phi = 2\pi\nu[BS/\Phi_0 - N_{qh}].$$

(Aharonov-Bohm part + statistical part)

In this case I_{bs} is:

- sinusoidal in S , $\Delta S = e/n_0$, exactly as in the $\nu = 1$ case,
- sinusoidal in B, except for occasional jumps happening when the number N_{qh} increases by one.

• This is the manifestation of the **statistical interaction among the quasi-holes flowing along the edge with those ones localized in the bulk**.

- High temperature $\Rightarrow$ multiple reflections.

  The back-scattered waves wind the cell several times before leaving the cell through one of the contacts.

- $$I_{bs} = \sum_{n=0}^{\infty} I_0^n \cos n(\phi + \alpha_0),$$

  ϕ given by the previous formula, α_0 (independent of B and area) coming from the phases of the tunnelling amplitudes of the two point contacts.

- Resonances at the values $\phi + \frac{2\pi N_{qh}}{m} = 2\pi l$, l integer, separated each other by areas $\Delta S = e/n_0$

Morally: in the limit of **strong** back-scattering the cell becomes a quantum dot, having a quantized number of electrons. **Along certain lines** in $B - S$ plane we have RESONANCES, where the energies of the dot with N electrons and $N + 1$ electrons are **degenerate**.

- This argument is based on the distinction:
  - **edge** quasi-holes $\longrightarrow$ flowing from one contact to another
  - **bulk** quasi-holes $\longrightarrow$ localized in the cell.
- In the experiments, the number N_{qh} of quasi-particles enclosed in the cell fluctuates with a certain time scale τ_0. As a consequence, the back-scattered current becomes time-dependent. Its noise in the limit of weak backscattering is:

$$[\langle I_{bs}^2 \rangle - \langle I_{bs} \rangle^2]_{\omega=0} \propto \langle I_{bs} \rangle^2 \tau_0 .$$

INTEGER Q.H.E.:

Experimentally, I_{bs} oscillates both in B and n:

$$\begin{cases} \text{Period } \Delta n \text{ independent of the integer filling factor } f. \\ \text{Period } \Delta B \propto \frac{1}{f} \end{cases}$$

• the "cell" can break into several regions of different phases. (Q.H. states with different ν, or compressible islands).

• Compressible regions complicates the experiments by adding indirect paths for tunnelling between the edges!

• The size of the compressible regions could vary by varying B or the back-gate voltage.

FRACTIONAL Q.H.E.:

Experimentally, I_{bs} oscillates both in B and n.

Data for $\nu = 1/3$ at the 2 constrictions, $\nu = 2/5$ at the enclosed island:

$$\begin{cases} \text{Period of oscill. in charge: } 2e \\ \text{Period of oscill. in magnetic flux: } 5\Phi_0. \end{cases}$$

This experiment is waiting for a satisfactory interpretation.