Breakdown and restoration of integrability in the Lieb-Liniger model

Giuseppe Menegoz

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Outline

1. 1-d bosons
2. Breaking integrability
3. Restoring integrability

Integrable systems

- Integrals of motion $\longleftrightarrow$ degrees of freedom

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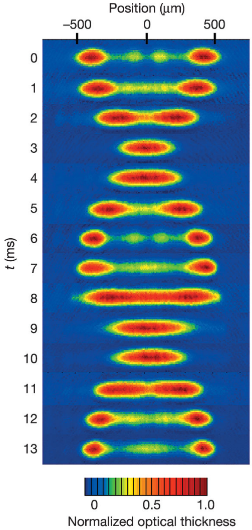

Kinoshita *et al.*, Nature **440** (2006), 900

1D Bosons

$$H = \frac{\hbar^2}{2m}\left[\sum_{i=1}^{N} -\nabla_i^2 + 2c\sum_{i<j}\delta(x_i - x_j)\right]$$

- Hardcore bosons on a line
- This system is integrable via Bethe Ansatz

$\frac{c}{n} \ll 1$

- Bogoliubov approximation

$\frac{c}{n} \gg 1$

- Fermion-like behavior

1D Bosons

How do we realize this model?

1D Bosons

Strongly elongated harmonic trap $\omega_r \gg \omega_z$

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- Integrability $\longrightarrow$ thermalization is prevented
- System is not really 1D $\longrightarrow$ thermalization is possible

3D system

$$\hat{H}_{3D} = \int d^3\mathbf{r} \left[\hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \hat{H}_r \right) \hat{\psi}(\mathbf{r}) + \right.$$
$$\left. + \frac{2\pi\hbar^2\alpha_s}{m} \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r}) \right]$$

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$$\hat{H}_r = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{m\omega_r^2}{2}(x^2 + y^2)$$

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$$\hat{\psi}(\mathbf{r}) = \frac{1}{\sqrt{L}} \sum_{nlk} \hat{a}_{nlk}\ \phi_{nl}(x, y)\ \mathrm{e}^{\imath kz}$$

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$$\hat{H}_r\ \phi_{nl} = (n+1)\hbar\omega_r \phi_{nl}(x,y) \quad \text{and} \quad L_z\phi_{nl} = l\phi_{nl}$$

Two-body collisions

$$(n, l)$$

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$$|l| = \mod (n, 2), \mod (n, 2) + 2, \ldots, n - 2, n$$

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- $|(0,0);(0,0)\rangle \longrightarrow |(0,0);(0,0)\rangle$

Two-body collisions

$$(n, l)$$

- $|(0,0);(0,0)\rangle \longrightarrow |(0,0);(0,0)\rangle$ NO THERMALIZATION (because of energy and momentum conservation)

Two-body collisions

$$(n, l)$$

- $|(0,0);(0,0)\rangle \longrightarrow |(0,0);(0,0)\rangle$ NO THERMALIZATION
- $|(0,0);(0,0)\rangle \longrightarrow |(0,l);(0,-l)\rangle$

Two-body collisions

Fermi Golden rule $T_{i\to f} = \frac{2\pi}{\hbar}\left|\langle f|\,\hat{H}\,|i\rangle\right|^2\rho$

$$\Gamma_{2b} \;\approx\; \frac{n_{1d}}{l_r^2}\times\alpha_s^2\times\frac{\hbar}{m l_r}\times \mathrm{e}^{-\frac{2\hbar\omega_r}{k_B T}}$$

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$\uparrow$

3d density

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cross section

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$$\Gamma_{2b} \approx \underbrace{\frac{n_{1d}}{l_r^2}}_{\text{3d density}} \times \overbrace{\alpha_s^2}^{\text{cross section}} \times \underbrace{\frac{\hbar}{m l_r}}_{\text{collision speed}} \times \mathrm{e}^{-\frac{2\hbar\omega_r}{k_B T}}$$

3d density collision speed

Two-body collisions

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$$\underset{\substack{\uparrow\\ \text{3d density}}}{\Gamma_{2b} \approx \frac{n_{1d}}{l_r^2}} \times \overset{\substack{\text{cross section}\\ \downarrow}}{\alpha_s^2} \times \underset{\substack{\uparrow\\ \text{collision speed}}}{\frac{\hbar}{m l_r}} \times \overset{\substack{\text{Boltzmann factor}\\ \downarrow}}{e^{-\frac{2\hbar\omega_r}{k_B T}}}$$

Two-body collisions

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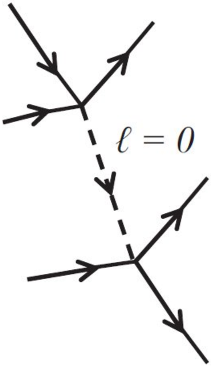
$$\Gamma_{2b} \;\approx\; \frac{n_{1d}}{l_r^2}\times\alpha_s^2\times\frac{\hbar}{ml_r}\times \mathrm{e}^{-\frac{2\hbar\omega_r}{k_BT}}$$

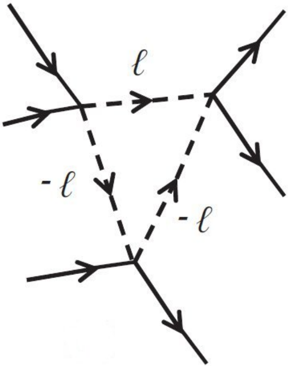
$$\Gamma_{2b}\sim 0.01\ s^{-1}$$

Three-body collisions



$$|(0,0);(0,0)\rangle \longrightarrow |(0,0);(2p,0)\rangle$$

Three-body collisions



$$|(0,0);(0,0)\rangle \longrightarrow |(n_1,l);(n_2,-l)\rangle$$

- substitute *ansatz* for $\hat{\psi}$ into $\hat{H}_{3D}$

- Integrate out x and y dependence (ϕ_{n0} are Laguerre polynomials)

- eliminate excited states

Effective Hamiltonian

$$\hat{H}_{1D} = \sum_k \frac{\hbar k^2}{2m} \hat{a}^\dagger_k \hat{a}_k + \frac{\hbar \omega_r \alpha_s}{L} \sum_{k,k',q} \hat{a}^\dagger_{k+q} \hat{a}^\dagger_{k'-q} \hat{a}_{k'} \hat{a}_k +$$
$$- \xi \frac{\hbar \omega_r \alpha_s^2}{2L^2} \sum_{\{k\}} \hat{a}^\dagger_{k_1} \hat{a}^\dagger_{k_2} \hat{a}^\dagger_{k_3} \hat{a}_{k_1} \hat{a}_{k_2} \hat{a}_{k_3}$$

three-body collision rates

$$|k_1, k_2, k_3\rangle = \hat{a}^\dagger_{k_1} \hat{a}^\dagger_{k_2} \hat{a}^\dagger_{k_3} |\mathrm{vac}\rangle \longrightarrow |k_1', k_2', k_3'\rangle$$

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$$\int dk'_1 \int dk'_2 \int dk'_3$$

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$$\int dk'_1 \int dk'_2 \int dk'_3$$

$$\Gamma_{k_1,k_2,k_3} = \zeta \frac{\omega_r \alpha_s^4}{L^2 l_r^2}$$

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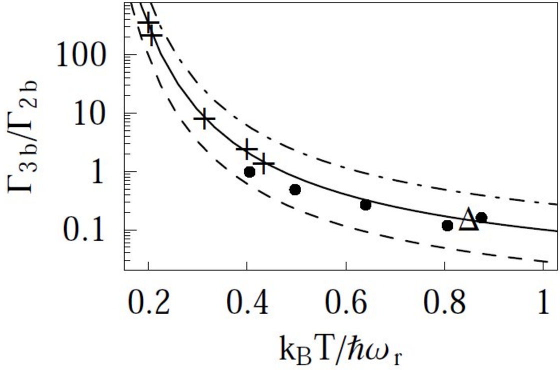
$$\int dk_1' \int dk_2' \int dk_3'$$

$$\Gamma_{k_1,k_2,k_3} = \zeta \frac{\omega_r \alpha_s^4}{L^2 l_r^2}$$

$$\Gamma_{3b} = \frac{1}{3!} \int dk \Gamma_{k_1,k_2,k_3} \prod (N f_k) = \frac{\zeta}{3!} \frac{\hbar}{\omega} \left(\frac{n_{1D}}{l_r^2} \right)^2 \alpha_s^4$$

No T dependence

Comparison



$$\frac{\Gamma_{3b}}{\Gamma_{2b}} \sim \left(\frac{n_{1D}\ \alpha_S^2}{l_r} \right)^2 e^{\frac{2\hbar\omega_r}{K_B T}}$$

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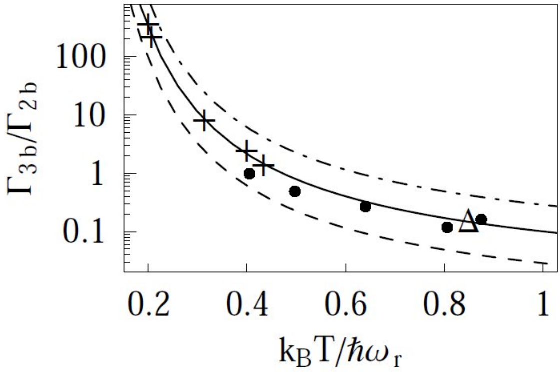

- Γ_{3b} can dominate in the appropriate experimental conditions
- In the weak interaction regime this ratio correspond also to the ratio between thermalization rates

Restored integrability

Strong interaction regime $\longrightarrow$ fermion-like behavior

$$g_3 = \frac{\left\langle \psi^\dagger \psi^\dagger \psi^\dagger \psi \psi \psi \right\rangle}{n_{1D}^3} \xrightarrow{c \to \infty} \frac{1}{c^6}$$

Low probability of a 3-body collision

Conclusions

- Radially confined 1D gases are never perfectly 1D, so radial motion can be virtually excited.
- These processes lead to thermalization.

- Integrability of confined 1D systems is not caused by the freeze out of 2-body collisions but is ensured by quantum correlations in the strongly interacting regime.

References

- I.E. Mazets, T. Schumm, J. Schmiedmayer, *Breakdown of integrability in a quasi-one-dimensional ultracold bosonic gas*, Phys. Rev. Lett. **100**, 210403 (2008)
- I.E. Mazets, J. Schmiedmayer, *Restoring integrability in one-dimensional quantum gases by two-particle correlations*, Phys. Rev. A **79**, 061603(R) (2009)