# Scaling of fluctuations for traffic flows in networks

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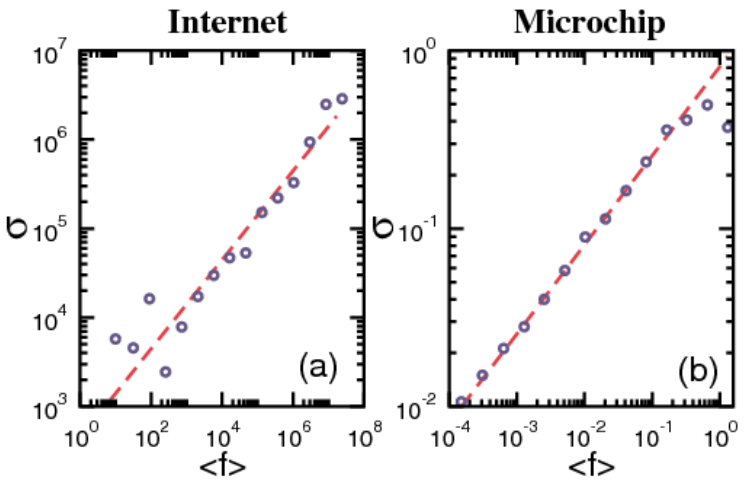
# Introduction and outlines

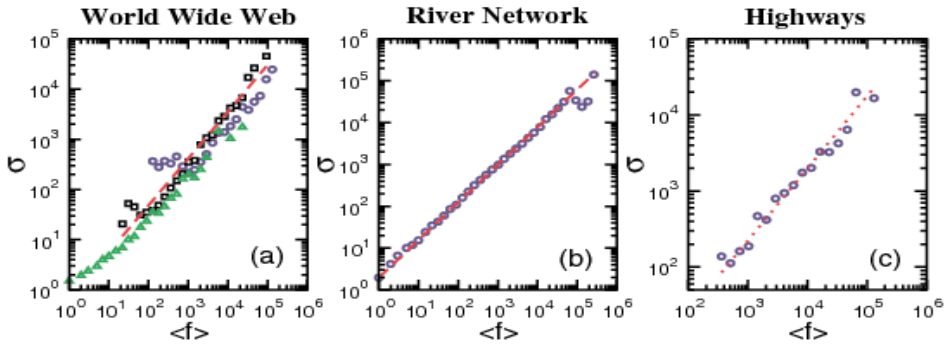
Statistical regularities emerging out of very complex phenomena, such as traffic in the Internet or in urban road networks, can be explained and discovered by methods of statistical mechanics.

- Different forms of scalings in traffic flows
- Normal diffusion for normal scaling
- Competition between internal and external fluctuations
- Scaling and queues formation
- Conclusions

# Scalings (I)

De Menezes and Barabasi PRL 92 (2004)



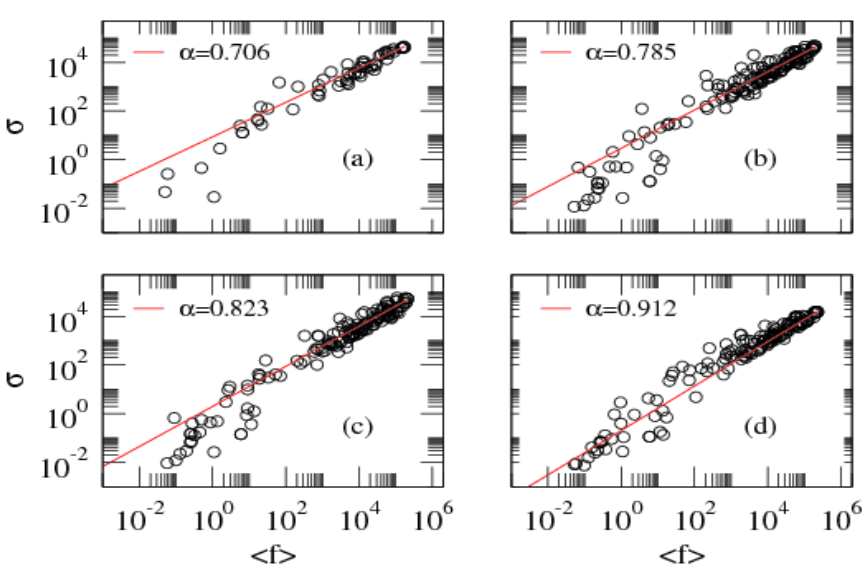

$\sigma \sim \langle f \rangle^{1/2}$

$\sigma \sim \langle f \rangle$

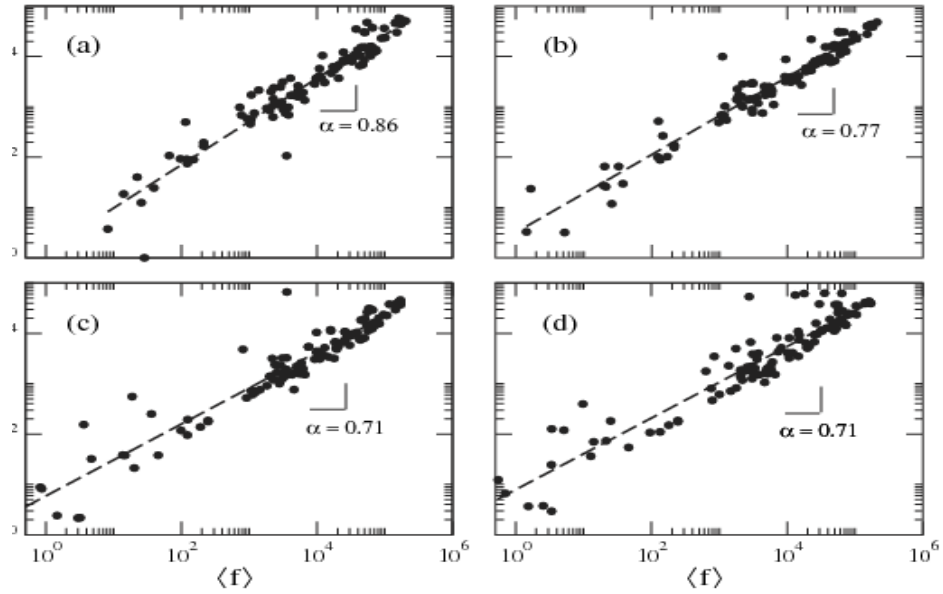
Are there two classes of universality?

# Scalings (II)

Meloni et al. PRL 100 (2008)




Duch and Arenas PRL 96 (2006)




Abilene interfaces

In general $\sigma \sim \langle f \rangle^{\alpha}$

# Normal diffusion for normal scaling

- M non-interacting random walkers onto a network of N nodes

- Poisson distribution at each node:
  $P(n) = e^{-c}\ c^n/n!$

- normal scaling $\sigma \sim \langle n \rangle^{1/2}$

# Competition between internal and external fluctuations

De Menezes and Barabasi PRL 92 (2004)

- M varies in [M−DM, M+DM] with uniform probability

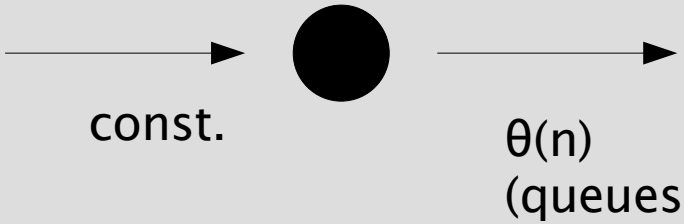
- Convolution of Poisson processes

- $\sigma^2 = \langle n \rangle + 2\,(DM/M)^2 \langle n \rangle^2$

# Inherent mechanism of queues formation

Duch and Arenas PRL 96 (2006)

- Random walks forming queues




- From detailed balance : $P(n) \propto x^n$

- $\sigma^2 = \langle n \rangle + \langle n \rangle^2$

# Queues formation ruled by external fluctuations

De Martino, Dall'Asta, Bianconi and Marsili PRE 79(R) (2009)

- Can we distinguish between the two processes? they are coupled....

- random walks created with rate p
  when they move, they are destroyed with rate μ

- $\sigma^2 = \langle n \rangle + \langle n \rangle^2$ with $\langle n \rangle = (p/\mu)/(1-p/\mu)$

- when $p > \mu$ the queue grows:
  $d\langle n \rangle/dt = p - \mu$ congestion!

# Conclusions and references

- For traffic flows in real networks, like The Internet, river and urban roads networks, ecc, different forms for the scaling of fluctuations at single node arise $\sigma \sim \langle f \rangle^{\alpha}$ with $\alpha$ between ½ and 1

-They can be explained by competition between external and internal fluctuations or by the interaction between particles (e.g. they form queues).

-The two mechanisms are coupled

References:
- De Menezes and Barabasi PRL 92 (2004)
- De Menezes and Barabasi PRL 93 (2004)
- Eisler, Kertesz,Yook and Barabasi EPL 69 (2005)
- Duch and Arenas PRL 96 (2006)
- Meloni et al. PRL 100 (2008)
- De Martino, Dall'asta, Bianconi and Marsili PRE 79(R) (2009)